The RF Photoinjector Gun: Microwave point-of-view

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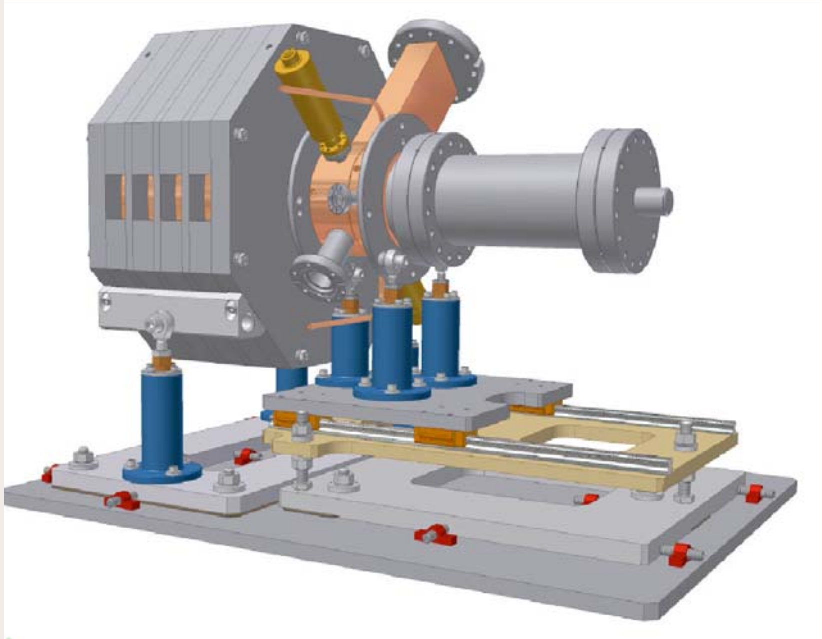
RF gun as microwave device

- Photoinjector guns are high gradient standing wave devices
  - Beam dynamics complex
  - Space-charge and RF are large effects
- Need to understand (a lot!):
  - Cavity resonances
  - Coupled cavity systems
  - Power dissipation
  - External coupling
  - Time-domain response
  - Measurement and tuning procedures
- Reference: Chapter 7 *Fundamentals of Beam Physics*

The photoinjector gun

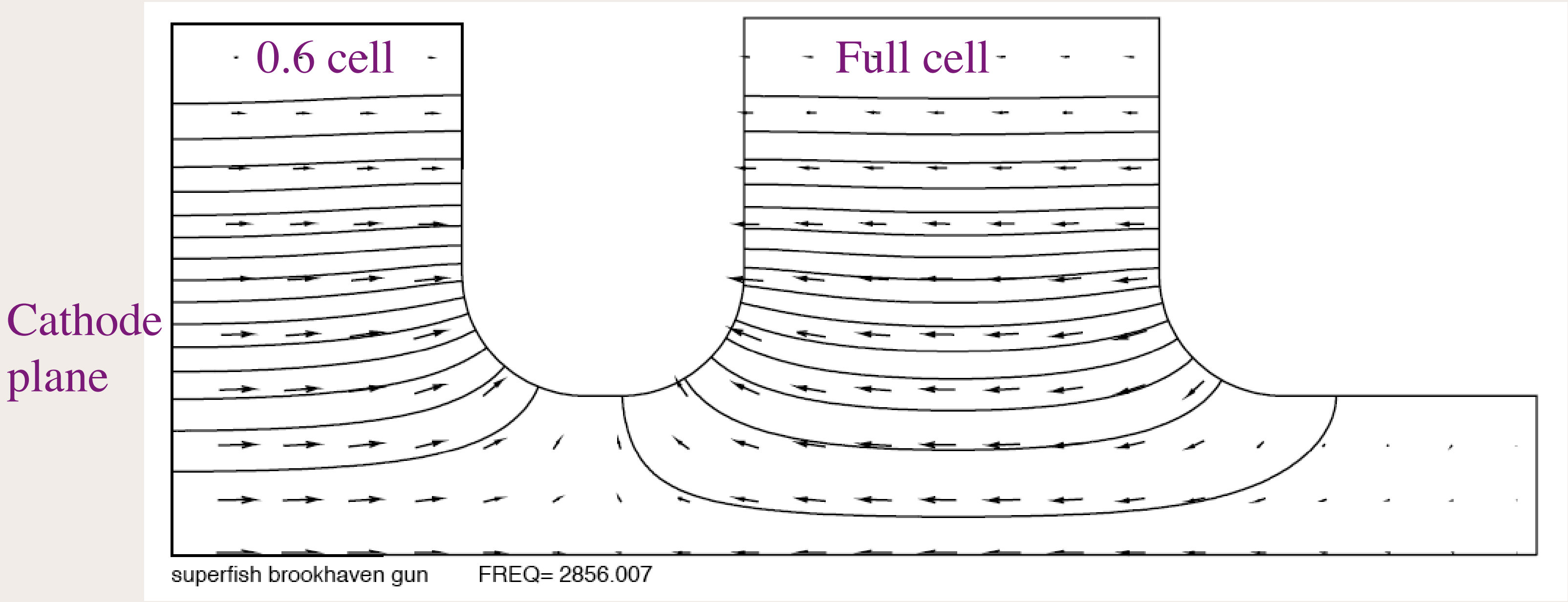
Neptune RF gun
with cathode plate removed

SPARC gun and solenoid



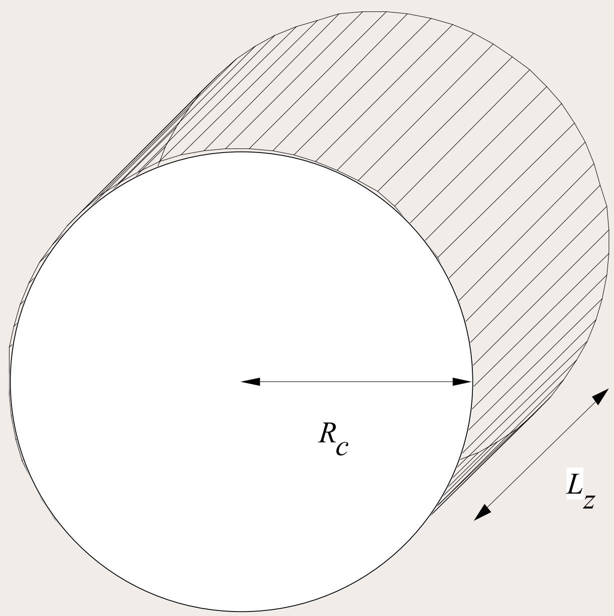
The 1.6 cell rf gun geometry

- Approximate with cylindrical symmetry



- π-mode, full (λ/2) cell with 0.6 cathode cell
- How to calculate resonant frequencies?

Cavity resonances: the pill-box model

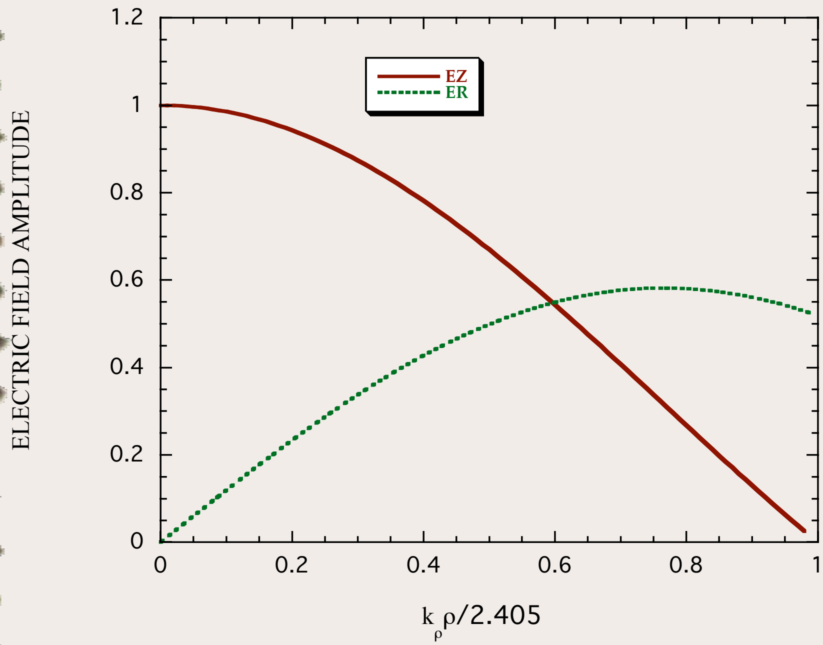


- Pill-box model approximates cylindrical cavities
- Resonances from Helmholtz equation analysis

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) - k_{z,n}^2 + \frac{\omega^2}{c^2}\right]\tilde{R} = 0$$

- Look for TM (axial field) modes
- Conducting BCs

Pill-box fields



- No longitudinal dependence in fundamental

$$k_{z,0} = 0 \qquad \omega_{0,1} \cong \frac{2.405c}{R_c}$$

- Electromagnetic fields

$$E_z(\rho) = E_0 J_0(k_{\rho,0}\rho)$$

$$H_\phi(\rho) = \frac{\omega\varepsilon_0}{k_{\rho,0}} E_0 J_1(k_{\rho,0}\rho) = c\varepsilon_0 E_0 J_1(k_{\rho,0}\rho)$$

- Stored energy

$$U_{EM} = \tfrac{1}{4}\varepsilon_0 L_z E_0^2 \int_0^{R_C} \left[J_0^2(k_{\rho,0}\rho) + J_1^2(k_{\rho,0}\rho)\right]\rho d\rho$$

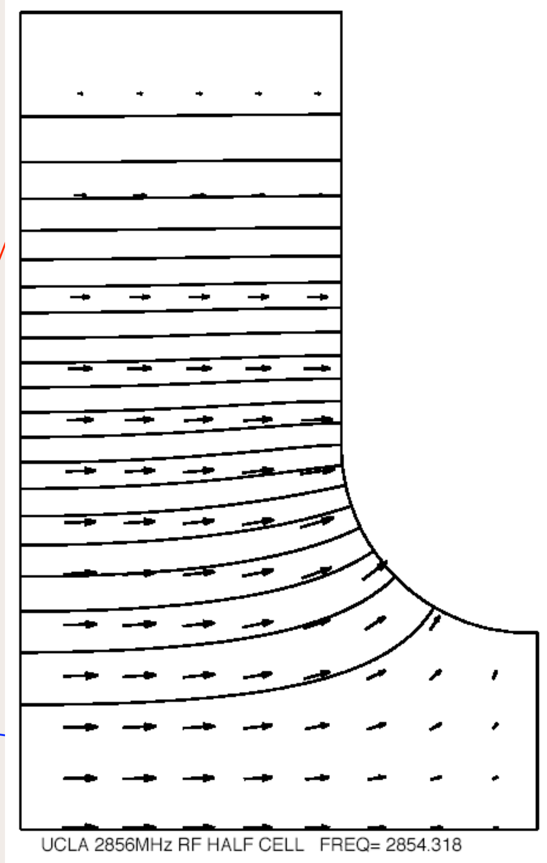
$$= \tfrac{1}{2}\varepsilon_0 L_z E_0^2 R_c^2 J_1^2(k_{\rho,0}R_C)$$

A circuit-model view

- Lumped circuit elements may be assigned: *L*, *C*, and *R*.
- Resonant frequency

$$\omega \cong \frac{1}{\sqrt{LC}}$$

- Tuning by changing inductance, capacitance
- Power dissipation by surface current (*H*)



Contours of constant flux in 0.6 cell of gun

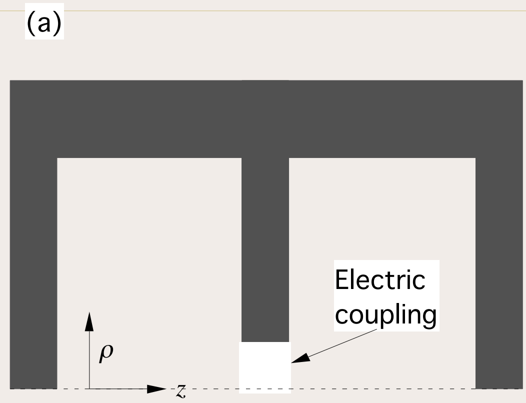
Cavity shape and fields

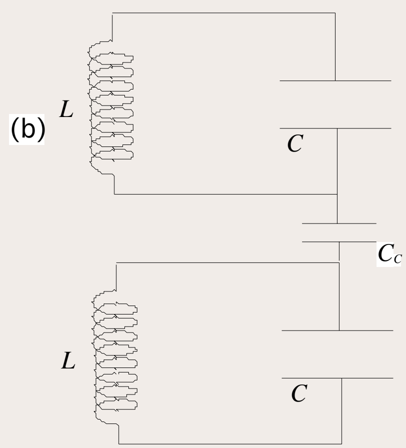
- Irises needed for beam passage and *RF coupling*
- Fields *near axis* (in iris region) may be better represented by spatial harmonics

$$E_z(\rho, z, t) = E_0 \,\mathrm{Im} \sum_{n=-\infty}^{\infty} a_n \exp\left[i\left(k_{n,z} z - \omega t\right)\right] I_0\left[k_{\rho,n} \rho\right] \qquad k_{\rho,n} = \sqrt{k_{n,z}^2 - \left(\omega / c\right)^2}$$

- Higher (no speed of light) harmonics have nonlinear (modified Bessel function) dependence on ρ.
  - Energy spread
  - Nonlinear transverse RF forces
- Avoid re-entrant nose-cones, etc. Circle-arc sections are close to optimum.

Cavity coupling: simple model





- Circuit model allows simple derivation of mode frequencies
- Ignore resistance in walls (does not affect frequency much)
- Electric coupling is *capacitive*

$$\frac{d^2 I_1}{dt^2} + \omega_0^2(1-\kappa_c)I_1 = -\kappa_c \omega_0^2 I_2$$

$$\frac{d^2 I_2}{dt^2} + \omega_0^2(1-\kappa_c)I_2 = -\kappa_c \omega_0^2 I_1$$

- Off-axis slots can give magnetic coupling (effectively reverses sign in κ)
- Solve eigenvalue/eigenmode equations

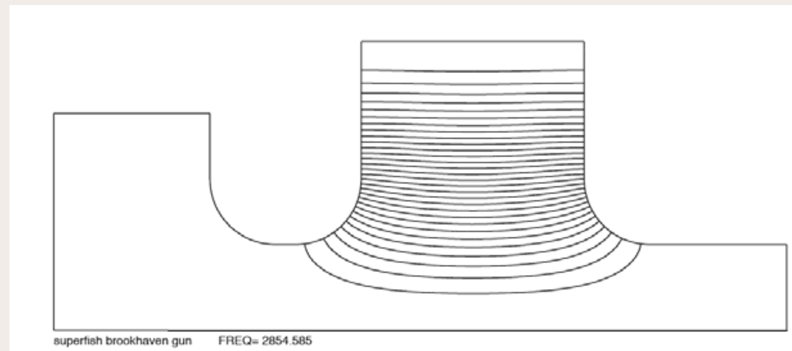
$$\begin{bmatrix} \omega_0^2(1-\kappa_c)-\omega^2 & \kappa_c\omega_0^2 \\ \kappa_c\omega_0^2 & \omega_0^2(1-\kappa_c)-\omega^2 \end{bmatrix}\mathbf{i} = 0$$

Coupled cavity modes

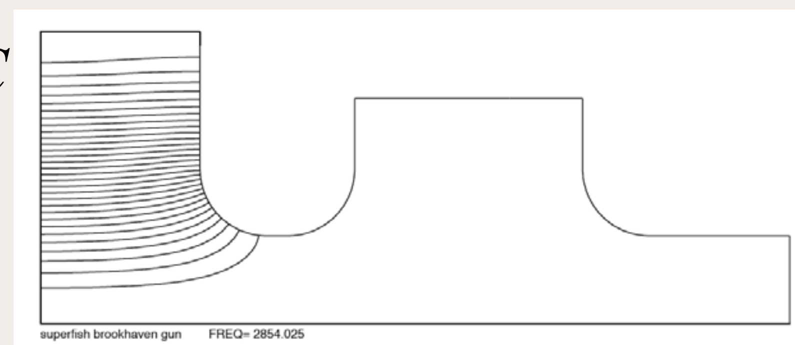
- Secular equation $\omega^4 - 2\omega_0^2(1-\kappa_c)\omega^2 + \omega_0^4(1-2\kappa_c) = 0$
- Two eigenvalues $\omega = \omega_0$ (0 - mode) and $\omega = \omega_0\sqrt{1+2\kappa_c}$ (π - mode)
- Corresponding to eigenvectors $\mathbf{i} = \frac{1}{\sqrt{2}}\begin{pmatrix} \pm 1 \\ 1 \end{pmatrix}$
- Lower frequency when cells are in phase (0-mode)
- The higher frequency mode is the π-mode, where the excitation is 180 degrees out of phase in the two cavity cells
  - Higher frequency due to less near-axis fields
  - Lower capacitance

Finding frequencies of cells

- Real geometry, use SUPERFISH simulation
- In simulation detune the opposing cell
- 0.6 cell has frequency of 2854.01 MHz
- Full cell is slightly higher: 2854.60 MHz due to lower *C*
- These are the geometries which produce “field” balance



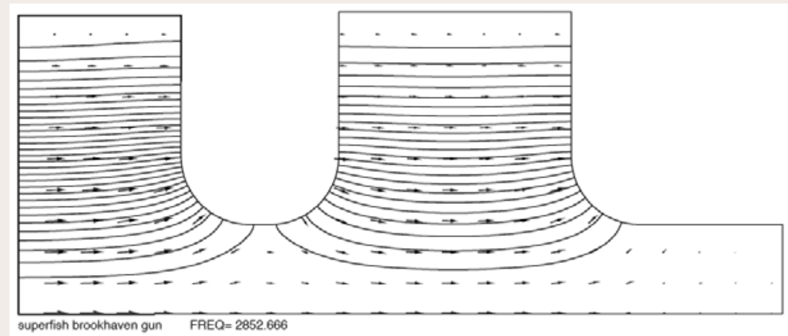
Full cell

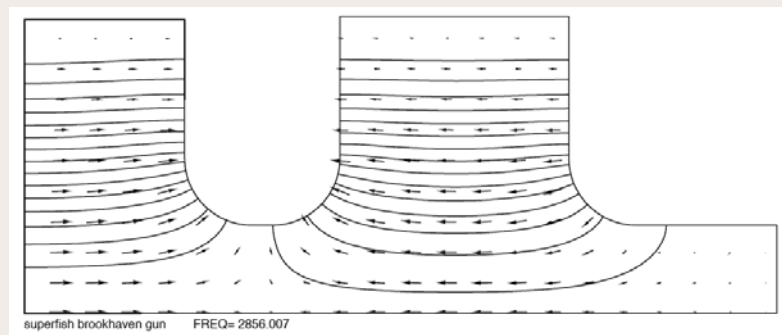


0.6 cell

The coupled mode frequencies

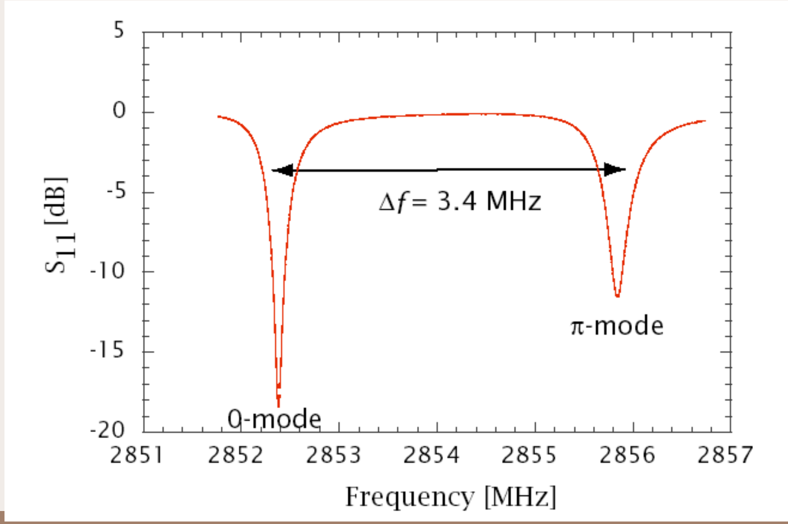
- The π-mode frequency is 2856.0 MHz as expected
- This mode needs to be balanced (equal fields)
- Zero mode frequency is 2852.66 MHz. Note that field arrows do not reverse
- The frequency separation is ~3.34 MHz according to calculation
  - This is sensitive to the iris geometry as built





Measurement and tuning of frequencies

- Frequency response can be measured on a network analyzer
  - *Calibrated* phase and amplitude response, calibrated frequency
- Two measurements: S_{11} (reflection) S_{21} (transmission)
- Resonance frequencies of individual cells and coupled modes
  - Full cell. Remove cathode to detune 0.6 cell.
  - 0.6 cell. S_{11} through (detuning) probe in full cell.
- Tuning via Slater's theorem/circuit model guide $\frac{\delta\omega_0}{\omega_0} = \frac{\delta V_c}{U_{EM}}\left[\frac{1}{2}\varepsilon_0\vec{E}^2 - \frac{1}{2}\mu_0\vec{H}^2\right]$

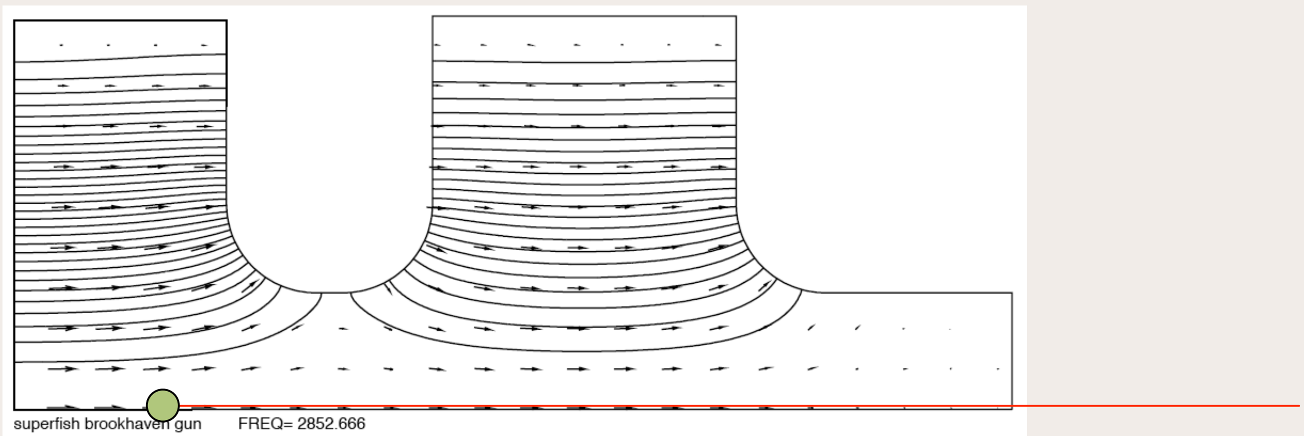


Measurement of fields

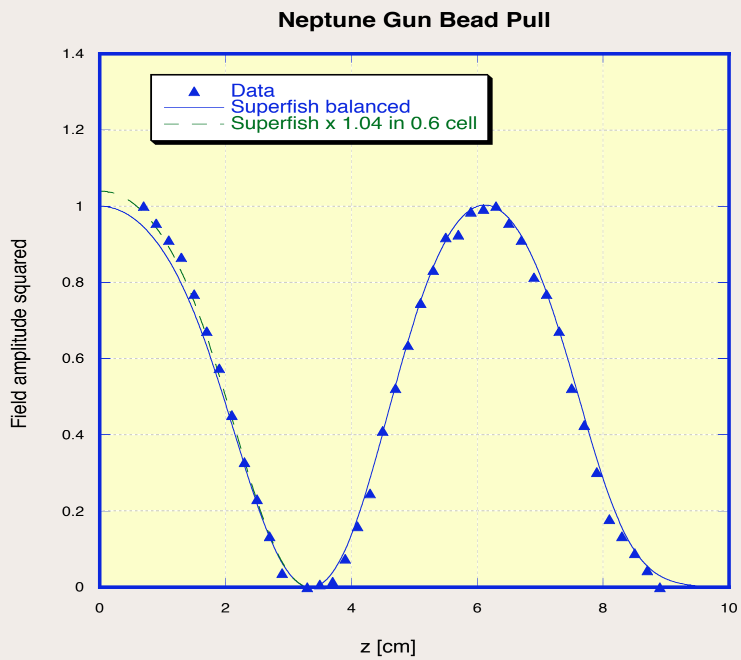
- Use so-called "bead-pull" technique
  - "Bead drop in gun, beam tube pointed upwards…
- Metallic or dielectric bead (on optical fiber)
- Metallic bead on-axis gives negative frequency shift (electric field energy displaced)
  - No magnetic effects on-axis for accelerating mode

$$|E_z| \propto \sqrt{-\delta\omega}$$

- More complex if one has magnetic fields (deflector)

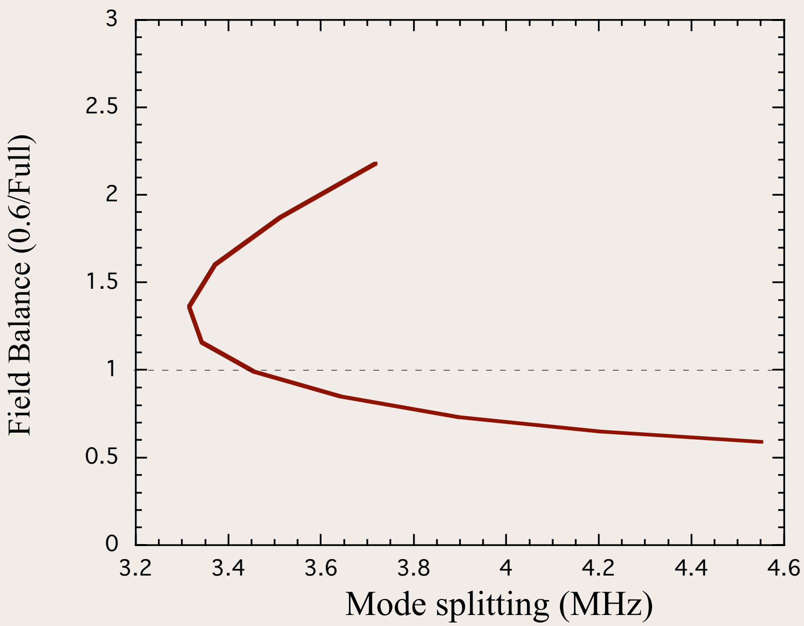


Field balance and mode separation

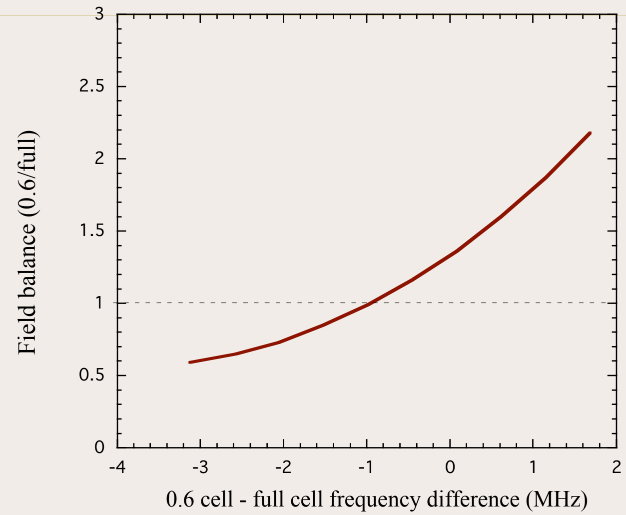


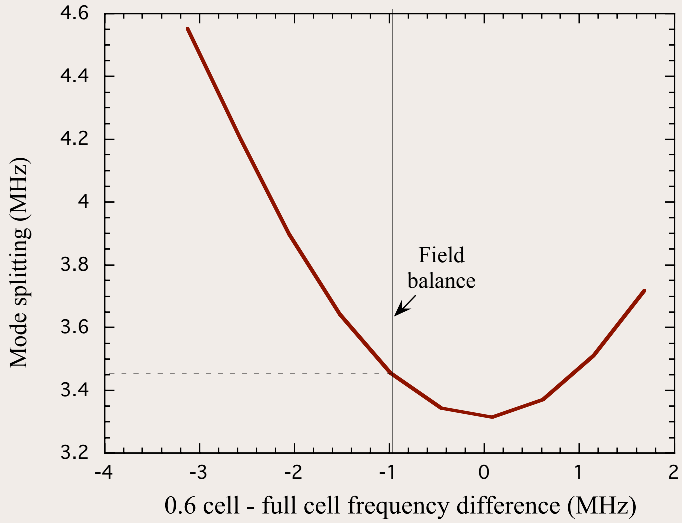
New Neptune gun field balance from bead pull

- Bead-pull technique is invasive
- Calibrate mode separation
  - Slightly different than coupled cavity model…
- Field balance possible in fast cathode swap



Other parameterizations

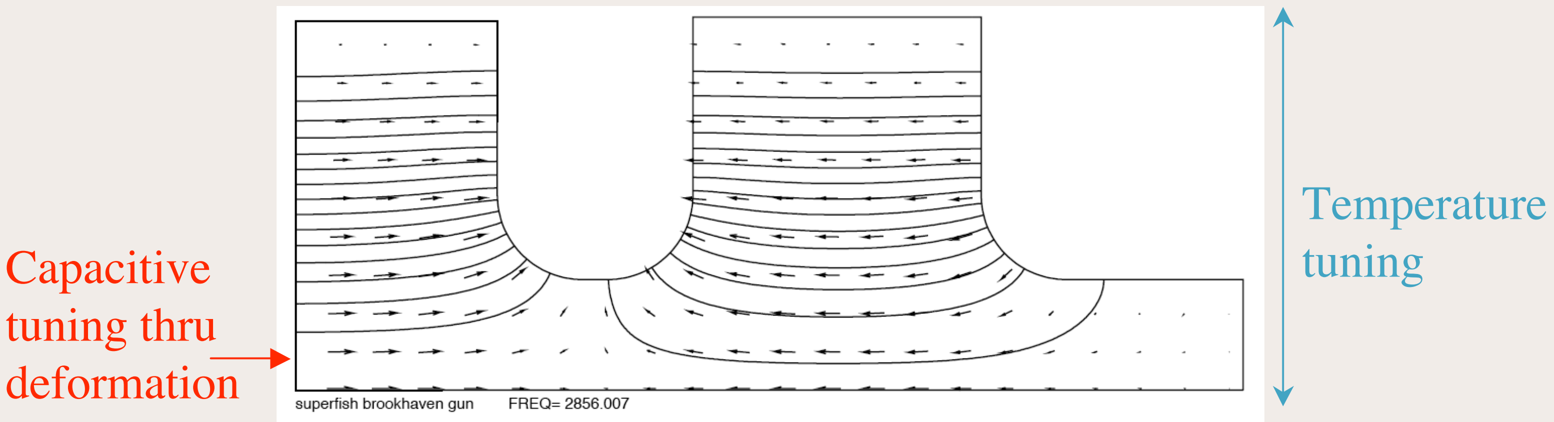




- Fundamental problem: the field balance is a double valued function of the mode splitting
- Full cell frequency is tuned by insertable (magnetic) tuners (in=higher)
- If you find minimum splitting, make full cell higher in frequency

Final tuning

- Do not want the full-cell tuners re-entrant (breakdown)
- Cathode deformation using “tuning nut”
- Temperature tuning=44 kHz/degree
- Almost perfect balance between atmospheric index and 20° C above room temperature operation



Power dissipation in walls

- Wave equation in conductors, harmonic solution

$$\left[\vec{\nabla}^2 - \mu_0\sigma_c\frac{\partial}{\partial t} - \mu_0\varepsilon\frac{\partial^2}{\partial t^2}\right]\begin{Bmatrix}\vec{E}\\ \vec{B}\end{Bmatrix} = 0 \qquad -k^2 + i\omega\mu_0\sigma_c + \mu_0\varepsilon\omega^2 = 0$$

- Complex wavenumber into wall normal, gives skin-depth

$$k = \sqrt{\frac{\omega\mu_0\sigma_c}{2}}(1+i) \qquad \delta_s \equiv [\operatorname{Im} k]^{-1} = \sqrt{\frac{2}{\omega\mu_0\sigma_c}}$$

- Power is lost in a narrow layer (skin-depth) of the wall by surface current excitation $K_s = \left|\vec{H}_{\parallel}\right| = \mu_0\left|\vec{B}_{\parallel}\right|$

$$\frac{dP}{dA} = -\frac{K_s^2}{4\delta_s\sigma_c} = -\frac{K_s^2}{4}\sqrt{\frac{\omega\mu_0}{2\sigma_c}} = -\frac{K_s^2}{2}R_s, \qquad R_s \equiv \frac{1}{2}\sqrt{\frac{\omega\mu_0}{2\sigma_c}} \quad \text{Surface resistivity}$$

Power dissipation and Q

- Total power dissipated in walls (pill-box example)

$$\langle P \rangle = \pi R_s \left(c\varepsilon_0 E_0 \right)^2 \left[R_c L_z J_1^2\left(k_{\rho,0} R_c\right) + 2\int_0^{R_c} J_1^2\left(k_{\rho,0}\rho\right)\rho d\rho \right]$$

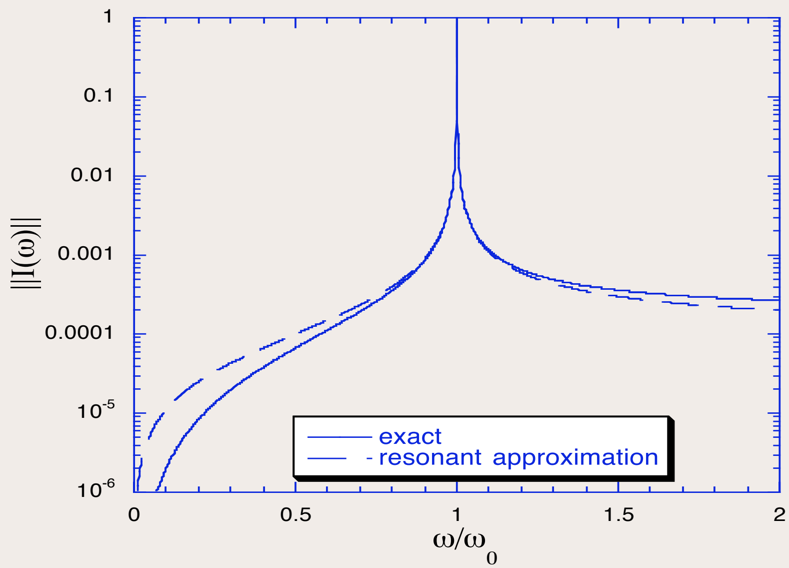
$$= \pi R_s \left(c\varepsilon_0 E_0 \right)^2 R_c J_1^2\left(k_{\rho,0} R_c\right)\left[L_z + R_c \right]$$

- Internal quality factor (pill box)

$$Q \equiv \frac{\omega U_{EM}}{\langle P \rangle} = \frac{Z_0}{2R_s} \frac{2.405 L_z}{\left(R_c + L_z\right)} \qquad Z_0 = 377\Omega$$

- For ~2856 MHz (S-band), Q~12,000
- Other useful interpretations of Q
  - Exponential response in time domain
  - Frequency response half-width

Frequency domain picture



$$Q = \frac{\omega}{\Delta\omega_{1/2}} = \frac{L}{R}$$

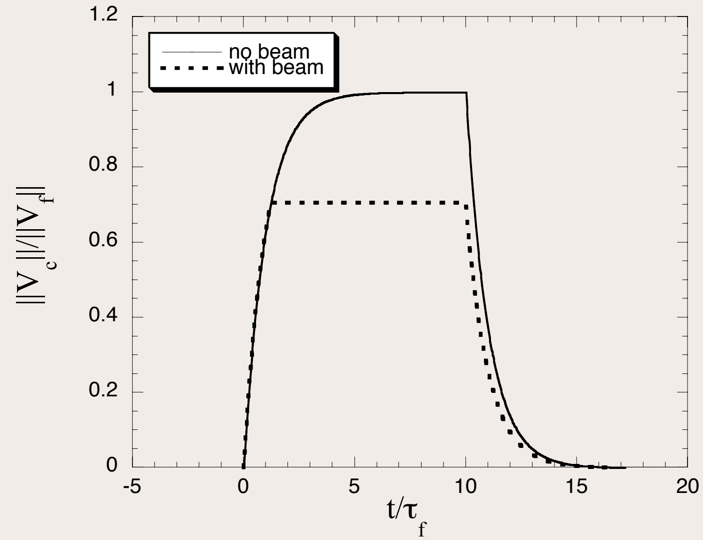
- Resonant width proportional to inverse of power loss
- Allows definition of lumped resistance
- Can measure with network analyzer
- Be careful about external coupling

Cavity filling, emptying, and external coupling

- Exponential response of cavity voltage

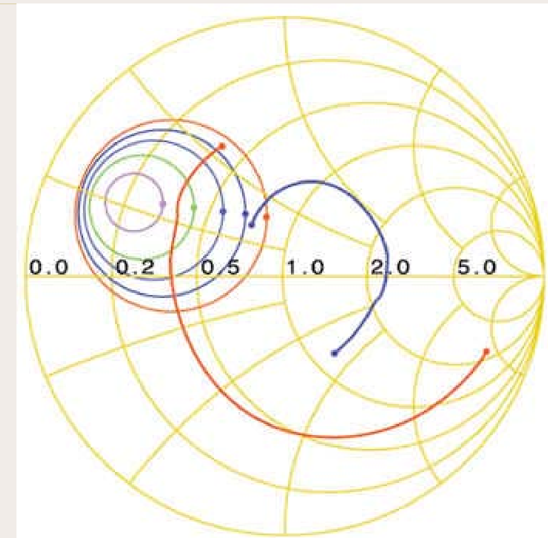
$$V_c(t) \cong \frac{2\beta_c}{1+\beta_c} V_F \sin\left(\left[\omega_0(t-t_0)\right]\right)\left[1-\exp\left(-\frac{\omega_0}{2Q_L}(t-t_0)\right)\right]$$

- Controlled by *loaded* Q_L
- Energy extraction by
  - Radiation into waveguide
  - Beam acceleration (high average current systems)



VSWR and β

- Measure reverse and forward voltage
  - Time domain
  - NWA Smith chart

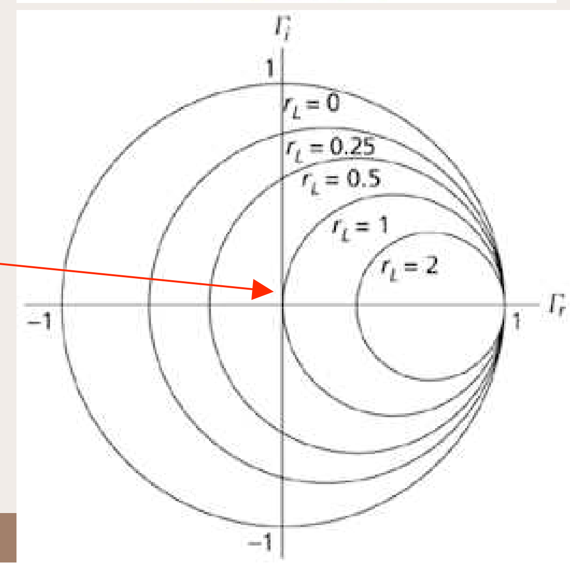


- Calculate VSWR

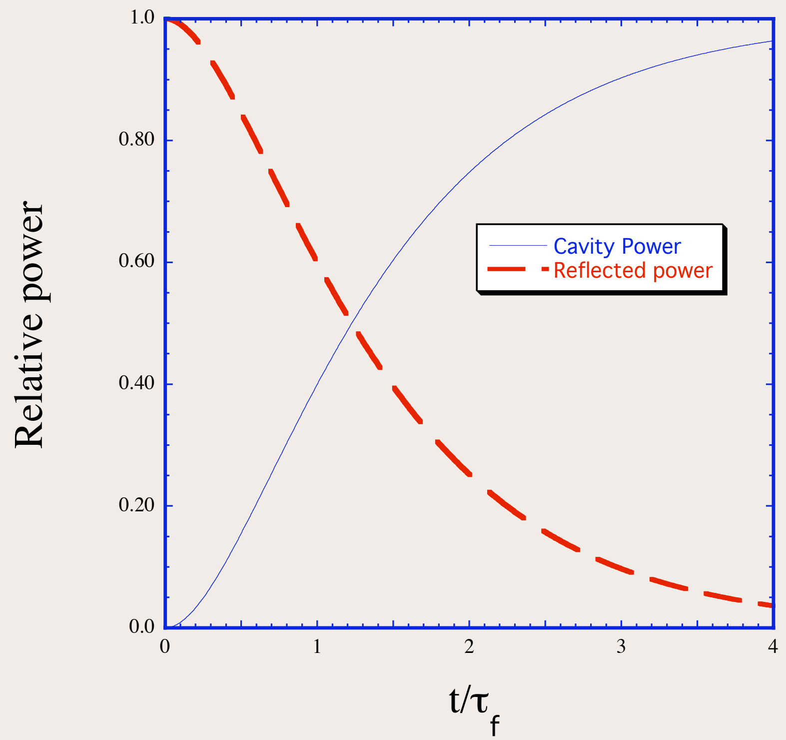
$$VSWR \equiv \frac{\|V_F\| + \|V_R\|}{\|V_F\| - \|V_R\|} = \begin{cases} \beta_c, & \beta_c > 1 \\ \beta_c^{-1}, & \beta_c < 1. \end{cases}$$

- We want critical coupling $\beta_c = 1$
- This is r_c=1 on lower right
- Loaded Q

$$Q_L = \frac{Q}{1+\beta_c} = \frac{Q}{2}$$



Temporal response of the cavity



- Standing wave cavity fills exponentially

$$E \propto 1 - \exp\left(-\frac{\omega t}{2Q}\right) \propto 1 - \exp\left(-\frac{t}{\tau_f}\right)$$

- Gradual *matching* of reflected and radiate power (E^2) from input coupler
  - Reflected wave from input coupler=re-radiated wave
- In steady-state, all power goes into cavity (critical coupling) so reflected power is eventually cancelled

Voltage, power and acceleration

- The square of the cavity voltage is proportional to the shunt impedance

$$P_c = \frac{\omega U_c}{Q_0} = \frac{L\|I_c\|^2}{\omega L / R} = \|I_c\|^2 R = \frac{\|V_c\|^2}{R} = \frac{V_c^2}{2R} = \frac{V_c^2}{Z_s}$$

- The accelerating field is The square of the cavity voltage is proportional to the shunt impedance per unit length

$$Z_s' = \frac{dP}{dz} \Big/ \left\langle E_0^2 \right\rangle$$

- The 1.6 cell gun S-band cavity has

$$Z_s' \cong 40 \text{ M}\Omega/\text{m}$$

The 1.6 cell RF gun

- The design power for the Neptune gun is *P*~6 MW
- The "accelerating" length of the gun is L_g=0.0845 m
- The average accelerating field is

$$E_0 = \sqrt{P\frac{Z'_s}{L_g}} = 53\ \text{MV/m}$$

- The peak on-axis accelerating field is ~ twice the average, or over 100 MV/m