Homework: Electron sources

Problems for Lecture 2.

1. Non-equilibrium solutions to the Vlasov equation are generally quite difficult, but it is possible to make a very powerful statement about solutions for cases in equilibrium, where $\frac{\partial f}{\partial t} = 0$. As an example of analysis of a *Vlasov equilibrium*, show that for a *time-independent* Hamiltonian, $H(\vec{x}, \vec{p})$,

$$f(\vec{x}, \vec{p}) = g\left[H(\vec{x}, \vec{p})\right],$$

is an equilibrium solution to the Vlasov equation, where g is *any* differentiable function of the Hamiltonian. What is the form of g for the example given in lecture?

2. (a) Prove, by direct differentiation, that the emittance $\varepsilon_{x,rms}$ is conserved under application of linear transverse forces $F_x \propto x$. (b) Prove, by direct differentiation, that the normalized emittance $\varepsilon_{n,x} \equiv \beta\gamma\varepsilon_{x,rms}$ is conserved under acceleration and linear transverse forces.

Problems for Lecture 3.

3. There are now "diode" sources with voltages as high as 500 kV, and so display relativistic effects. As a step for generalizing the Child-Langmuir law, solve for the first integral of the *relativistic* diode. This will require substituting the relativistically correct relationship for the velocity as a function of energy. Note: the second integral is difficult; if you are interested, numerical solution of the second integral is recommended.

4. In a Gaussian profile beam, with intensity proportional to $I(r) = I_{\max} \exp\left(-r^2/2\sigma_r^2\right)$, the cutoff of photoemission from longitudinal space charge occurs slowly, with only the middle being limited by the self-field to have no more surface charge density than $\sigma_{bm} = E_0 \sin\phi_0 / 4\pi$. Show that the emitted charge is degraded from the expected value by

$$\frac{Q_{\text{emitted}}}{Q_{\text{expected}}} = \frac{\sigma_{bm}}{\sigma_{b0}}\left[1 + \ln\left(\frac{\sigma_{b0}}{\sigma_{bm}}\right)\right], \quad \text{where } \sigma_{b0} = \frac{Q_{\text{expected}}}{2\pi\sigma_r^2} .$$