Lecture 4: Emittance Compensation

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Emittance minimization in the RF photoinjector

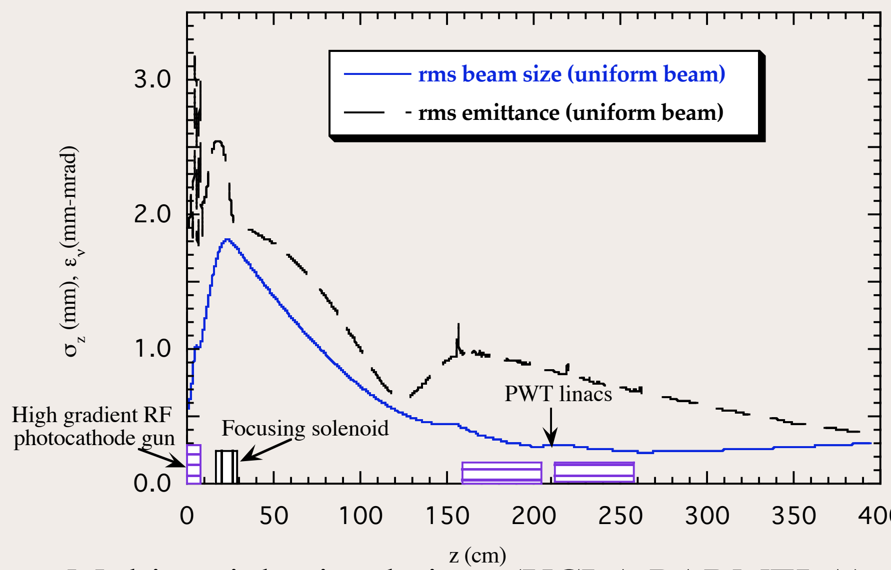
- Thermal emittance limit
  - Small transverse beam size
  - Avoid metal cathodes? $\varepsilon_{n,th} \approx \frac{1}{2}\sqrt{\frac{h\nu - W}{m_0 c^2}}\sigma_x \approx 5\times 10^{-4}\sigma_x(\mathrm{m})$
- RF emittance $\varepsilon_{n,RF} \approx k_{RF}\alpha_{RF}\left(k_{RF}\sigma_z\right)^2\sigma_x{}^2$
  - Small beam dimensions
  - Small acceleration field? Maybe not…
- Space charge emittance
  - K.J.Kim treatment is very discouraging

$$\varepsilon_{n,sc} \approx \frac{m_e c^2}{(2\pi)^2 e E_0}\frac{10I}{I_0(1+\frac{3}{5}A)} \qquad A = \frac{\sigma_x}{\sigma_z},\ I_0 = \frac{ec}{r_e}$$

$$\varepsilon_{n,sc} \approx 5\ \mathrm{mm\text{-}mrad}\ (I = 100\ \mathrm{A},\ E_0 = 100\ \mathrm{MV/m})$$

Space-charge emittance control?

- Kim model indicates monotonic emittance growth due to space-charge
- Multiparticle simulations at LLNL (Carlsten) show emittance oscillations, minimization possible: *Emittance compensation*
- Work extended by UCLA, INFN scientists to give analytical approach
- New high gradient design developed and understood
- Many new doors opened



Multiparticle simulations (UCLA PARMELA)
Showing emittance oscillations and minimization

Intense beam dynamics in photoinjector: a demanding problem

- Extremely large applied fields
  - Violent RF acceleration (0 to ~3E8 m/s in < 100 ps)
  - Large, possibly time-dependent external forces (rf and focusing solenoids)
- Very large self-fields
  - Longitudinal debunching (charge limit)
  - Radial oscillations (single component plasma)
- Optimization of beam handling with large parameter space and collective effects. Multiparticle simulations are invaluable aid, <u>but time-consuming</u>
- *Understanding* of non-equilibrium transport approached using rms envelope equations…

Transverse dynamics model

- After initial acceleration, space-charge field is mainly transverse (beam is long in rest frame).
- Force scales as γ^{-2} (cancellation of electric defocusing with magnetic focusing)
- Force dependent almost exclusively on local value of current density I/σ^2 (electric field simply from Gauss' law)
- Linear component of self-force most important. *We initially assume that the beam is nearly uniform in* r.
- The linear "slice" model…
- Extend linear model to include nonlinearities within slices
- Scaling of design physics with respect to charge, λ_{RF}

The rms envelope equation

- The rms envelope dynamics for a *cylindrically symmetric, non-accelerating, space-charge dominated* beam are described by a nonlinear differential equation

$$\sigma_x''(\zeta,z) + k_\beta^2 \sigma_r(\zeta,z) = \frac{r_e \lambda(\zeta)}{2\gamma^3 \sigma_x(\zeta,z)} + \frac{\varepsilon_{n,x}^2}{\gamma 2 \sigma_x^3(\zeta,z)}$$

- Separate DE for each slice (tagged by ζ), $\zeta = z - v_b t$
- Each slice has different current $I(\zeta) = \lambda(\zeta) v$
- External focusing measured by *betatron wave-number*

$$k_\beta = eB_z / 2p_0$$

- In solenoid, beam is rotating, so envelope coordinates are in rotating Larmor frame with same wave-number
- Rigid rotator equilibrium (Brillouin flow) depends on local value of current (line-charge density). "Pressure" forces negligible

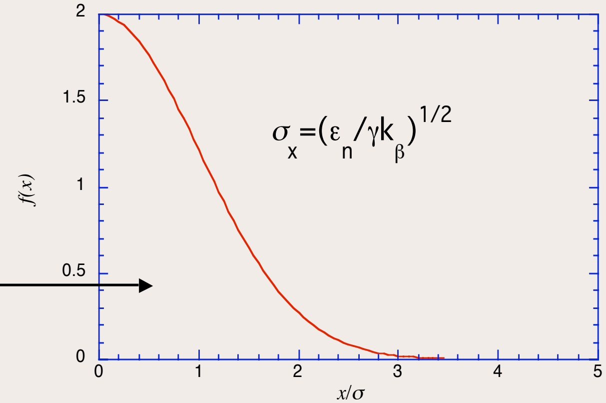
$$\sigma_{eq}(\zeta) = \frac{1}{k_\beta}\sqrt{\frac{r_e \lambda(\zeta)}{2\gamma^3}} \qquad r_e \lambda(\zeta) = I(\zeta)/I_0$$

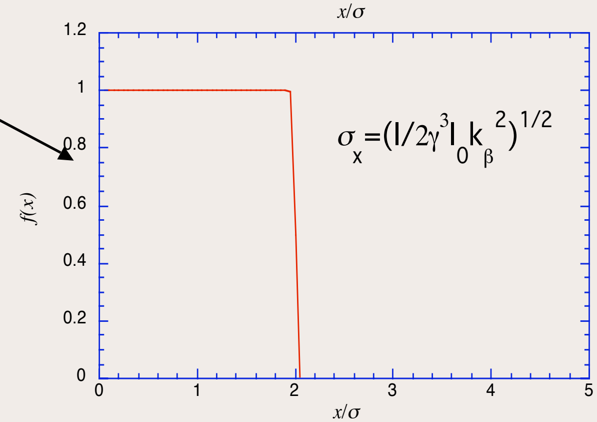
Equilibrium distributions and space charge dominated beams

- Maxwell-Vlasov equilibria have simple asymptotic forms, dependent on parameter

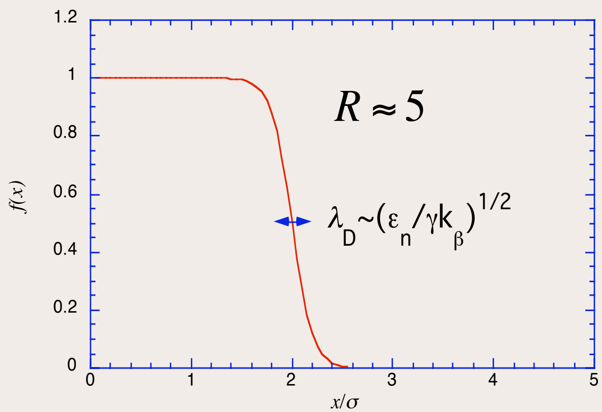
$$R \equiv I/2\gamma^2 k_\beta \varepsilon_n I_0$$

- Emittance dominated gaussian $R \ll 1$
- Space-charge dominated uniform $R >> 1$





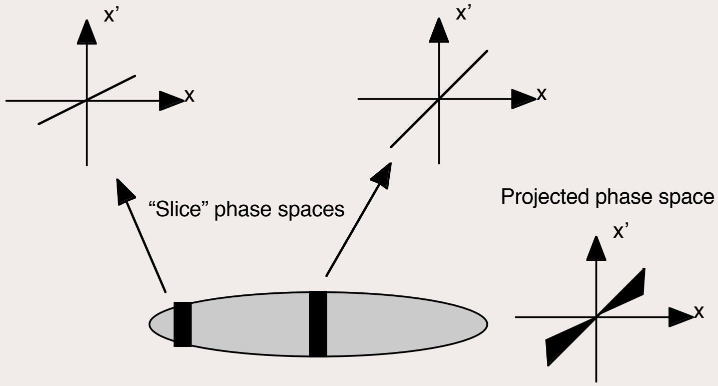
- Uniform beam approximation very useful



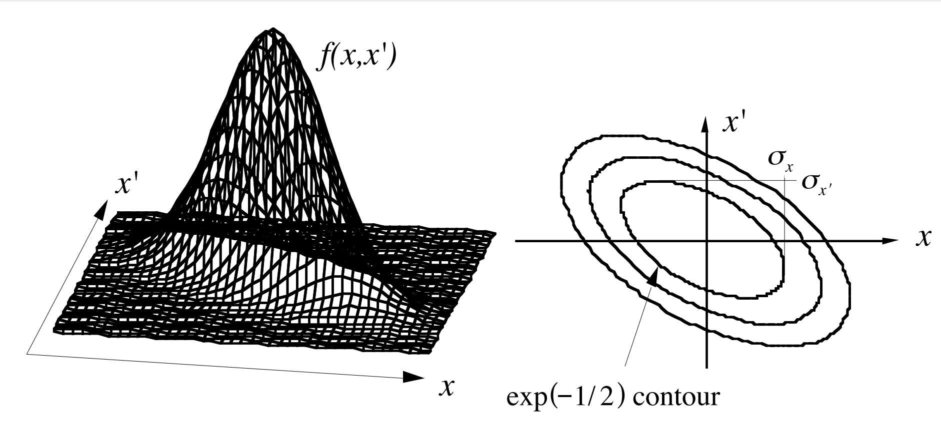
- Nominally uniform has Debye sheath
- High brightness photoinjector beams have $R > 1, \;\; \gamma \leq 250!$

The trace space model

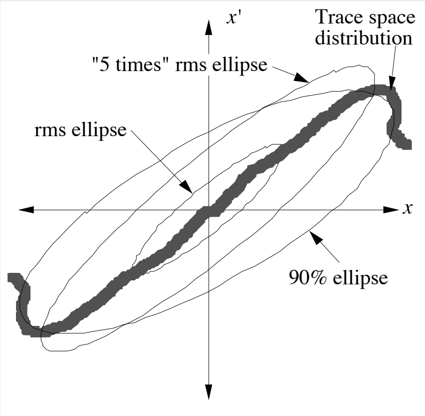
- Each ζ-slice component of the beam is a line in trace space.
- No thermal effects
- No nonlinearities (lines are straight!)



The model



Contrast with thermal trace space…



and nonlinear slice trace space

Envelope oscillations about equilibria

- Beam envelope is *non-equilibrium* problem, however
- Linearizing the rms envelope equation about its equilibria gives

$$\delta\sigma_x''(\zeta,z) + 2k_\beta^2\sigma_x(\zeta,z) = 0$$

Dependent on betatron wave-number, *not* local beam size or current

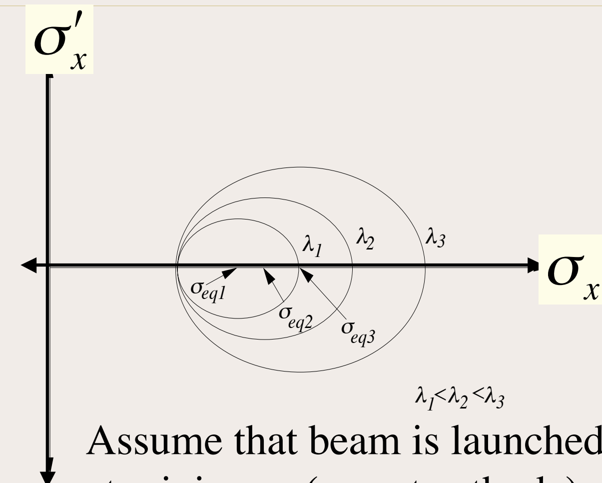
- Small amplitude envelope oscillations proceed at $2^{1/2}$ times the betatron frequency or *assuming uniform beam distribution*

$$k_{\text{env}} = \sqrt{2}k_\beta = \sqrt{\frac{4\pi r_e n_{b,eq}}{\gamma^3}} = k_p$$

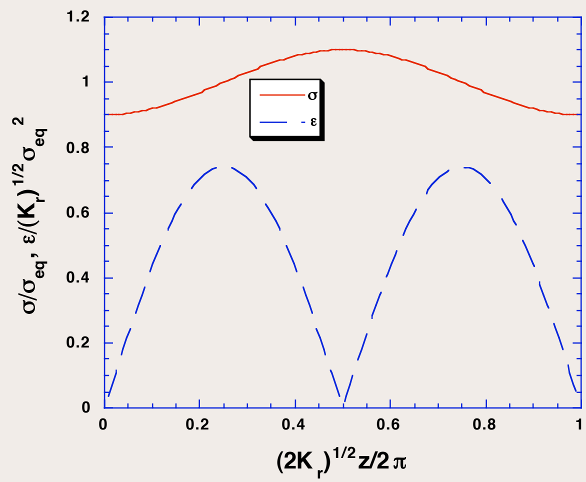
This is the *matched* relativistic plasma frequency

Phase space picture: coherent oscillations

- All oscillations of space-charge beam envelope proceed about
  - different equilibria,
  - with different amplitude
  - but at the *same frequency*
- Behavior leads to emittance oscillations…but not damping (yet)
- Qualitative explanation of "1st compensation", after gun, before linac…

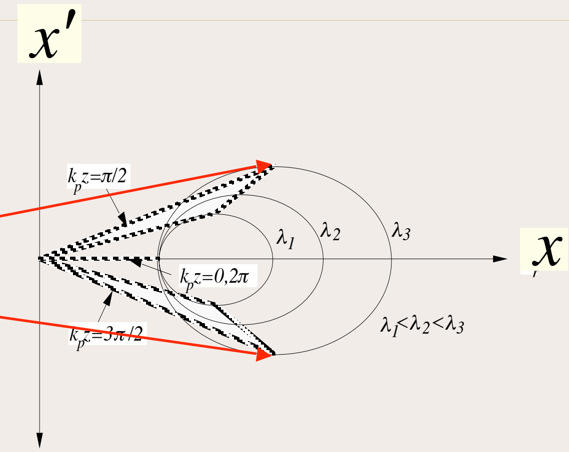


Assume that beam is launched at minimum (e.g. at cathode)



Small amplitude oscillation model

Phase space picture: coherent oscillations



- Emittance (area in phase space) is maximized at

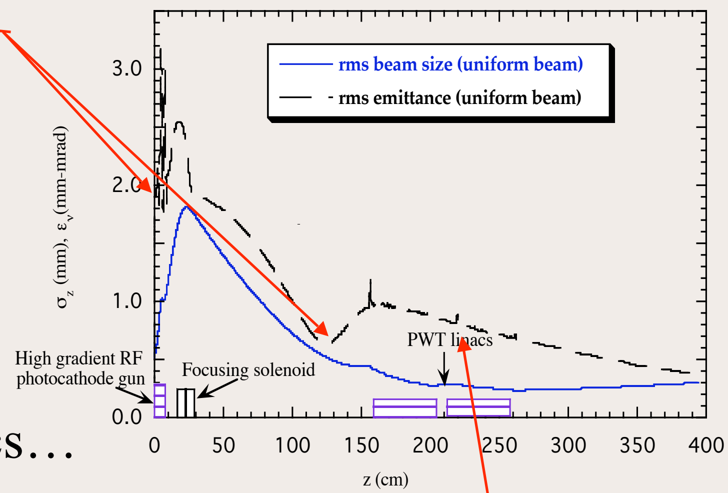
$$k_p z = \pi/2, 3\pi/2$$

- Emittance is locally minimized at

$$k_p z = 0, \pi, 2\pi$$

- the beam extrema!

- Fairly good agreement of simple model with much more complex beamline
- What about acceleration?
  - In the rf gun, in booster linacs…



Damping of ε...

Emittance damping: Beam envelope dynamics under acceleration

- Envelope equation (w/o emittance), with acceleration, RF focusing

$$\sigma_x''(\zeta,z) + \left(\frac{\gamma'}{\gamma(z)}\right)\sigma_x'(\zeta,z) + \frac{\eta}{8}\left(\frac{\gamma'}{\gamma(z)}\right)^2 \sigma_x(\zeta,z) = \frac{r_e\lambda(\zeta)}{2\gamma(z)^3\sigma_x(\zeta,z)}$$

$\eta \approx 1$ (rf or solenoid focusing)

$\gamma' = eE_0/m_0c^2$ (accel. "wavenumber")

- New particular solution - the "*invariant envelope*" (generalized Brillouin flow), slowly damping "fixed point"

$$\sigma_{inv}(\zeta,z) = \frac{1}{\gamma'}\sqrt{\frac{r_e\lambda(\zeta)}{(2+\eta)\gamma(z)}} \propto \gamma^{-1/2}$$

- Angle in phase space is *independent of current* $\theta = \frac{\sigma'_{inv}}{\sigma_{inv}} = -\frac{1}{2}\frac{\gamma'}{\gamma}$

- Corresponds *exactly* to entrance/exit kick (matching is naturally at waists)

$$\Delta x'_{RF} = -\frac{1}{2}\frac{\gamma'}{\gamma}x$$

- Matching beam to invariant envelope yields stable *linear* emittance compensation!

Envelope oscillations near invariant envelope, with acceleration

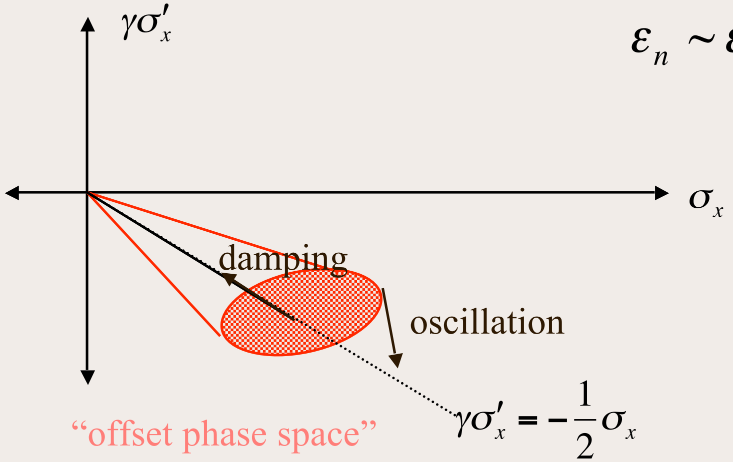
- Linearized envelope equation

$$\delta\sigma_x'' + \left(\frac{\gamma'}{\gamma}\right)\delta\sigma_x' + \frac{1+\eta}{4}\left(\frac{\gamma'}{\gamma}\right)^2 \delta\sigma_x = 0$$

- Homogenous solution (independent of current)

$$\delta\sigma_x = [\sigma_{x0} - \sigma_{inv}]\cos\left(\frac{\sqrt{1+\eta}}{2}\ln\left(\frac{\gamma(z)}{\gamma_0}\right)\right)$$

- Normalized, projected phase space area oscillates, seculary *damps* as offset phase space (conserved!) moves in…



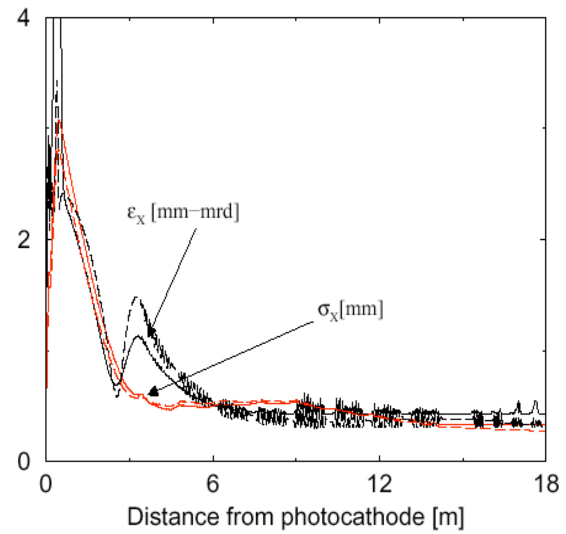
$$\varepsilon_n \sim \varepsilon_{offset}\sigma_{\text{inv}} \sim \gamma^{-1/2}$$

Oscillation (matched plasma) frequency damps with energy

$$k_p = \frac{d}{dz}\left(\frac{\sqrt{1+\eta}}{2}\ln(\gamma(z))\right) = \frac{\sqrt{1+\eta}}{2}\frac{\gamma'}{\gamma}$$

Validation of linear emittance compensation theory

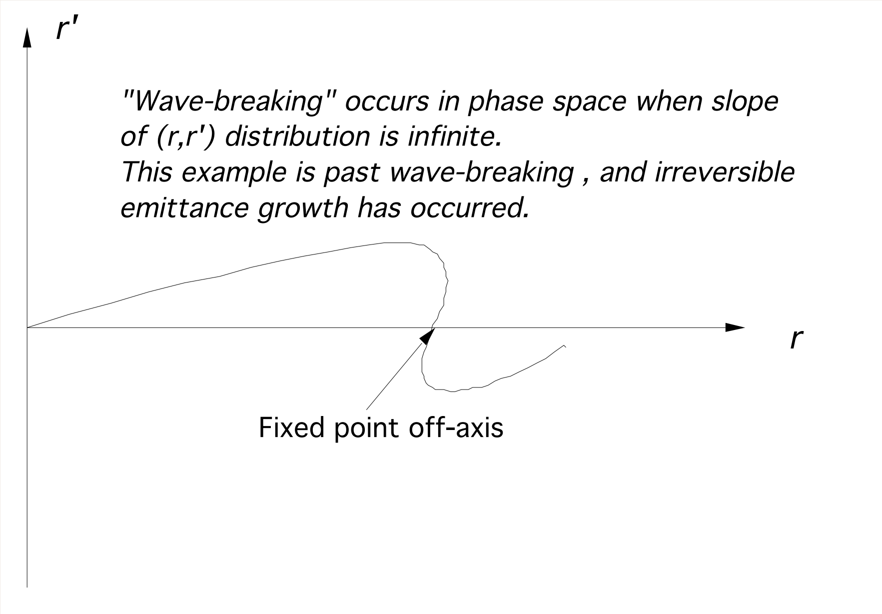
- Theory successfully describes “linear” emittance oscillations
  - “Slice” code (HOMDYN) developed that reproduce multiparticle simulations. Much faster! Ferrario will lecture on this..
  - LCLS photoinjector working point found with HOMDYN



Dash: HOMDYN
Solid: PARMELA

Nonlinear Emittance Growth

- Nonuniform beams lead to nonlinear fields and emitance growth
- It is well known from the heavy ion fusion community that propagation of non-uniform distributions in *equilibrium* leads to *irreversible* emittance growth (wave-breaking in phase space).



Fixed point is where space-charge force cancels applied (solenoid) force. It is in the middle of the Debye sheath region.

Non-equilibrium, nonlinear "slice" dynamics

- Matching of envelope to "invariant" envelope guarantees that we have linear emittance compensation; is it courting nonlinear emittance growth?
- Understanding obtained as before by:
  - Heuristic analysis
  - Computational models

Heuristic slab-model of non-equilibrium laminar flow

- Laminar flow=no trajectory crossing, no wavebreaking in phase space
- Consider first free expansion of slab (infinite in *y, z*) beam (very non-equilibrium)

$$n_b(x_0) = n_0 f(x_0),\ f(0) = 1$$

- Under laminar flow, the charge inside of a given electron is conserved, and one may mark trajectories from initial offset x_0. Equation of motion

$$x'' = k_p^2 F(x_0),\ F(x_0) = \int_0^{x_0} f(\tilde{x}_0) d\tilde{x}_0 = const.$$

Note, with normalization $F(x_0) \propto x$

Free-expansion of slab beam

- Solution for electron positions:

$$x(x_0) = x_0 + \frac{(k_p z)^2}{2} F(x_0)$$

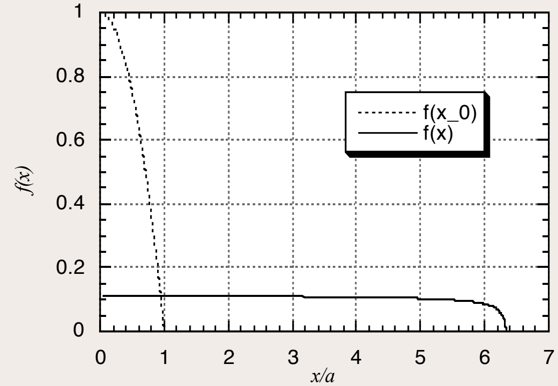
- Distribution becomes more linear in density with expansion

$$f(x(x_0)) = \frac{f(x_0)}{1 + \frac{(k_p z)^2}{2} f(x_0)} \Rightarrow \frac{2}{(k_p z)^2}$$

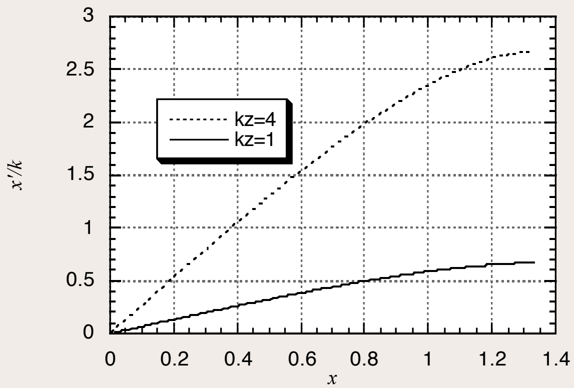
- Example case $f(x_0) = 1 - \left(\frac{x_0}{a}\right)^2$

- Wavebreaking will occur when final x is independent of initial x_0, $\frac{dx}{dx_0} = 0$
- In free-expanding slab, we have no wave-breaking for any profile

$$\frac{dx}{dx_0} = 1 + \frac{(k_p z)^2}{2} f(x_0) > 1 > 0$$



Initially parabolic profile becomes more uniform at $k_p z = 4$



Phase space profile becomes more linear for $k_p z >> 1$

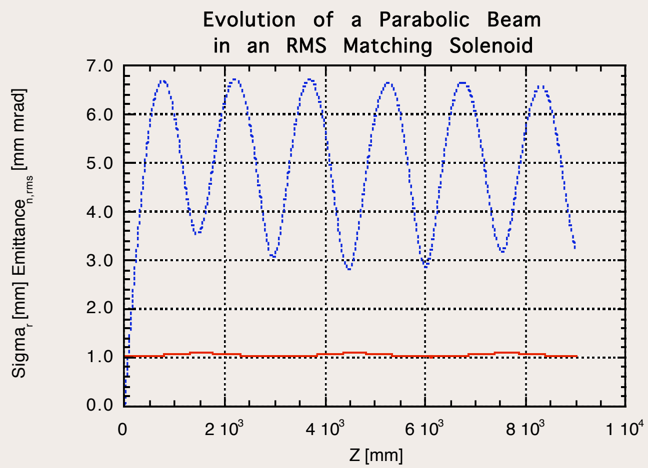
Slab-beam in a focusing channel

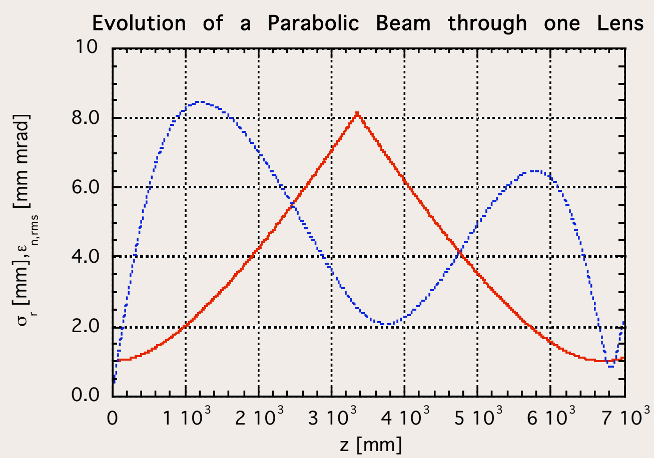
- Add uniform focusing to equation of motion,

$$x'' + k_\beta^2 x = k_p^2 F(x_0).$$

- Solution $x(x_0) = x_{eq}(x_0) + \left[x_0 - x_{eq}(x_0)\right]\cos(k_\beta z)$
  with $x_{eq}(x_0) = \frac{k_p^2}{k_\beta^2} F(x_0)$
- Wavebreaking occurs in this case for $f(x_0) = -\frac{k_\beta^2}{k_p^2} \frac{\cos(k_\beta z)}{2\sin^2\left(\frac{k_\beta z}{2}\right)}$
- For physically meaningful distributions, $f(x_0) \to 0$ smoothly, and wavebreaking occurs when $k_\beta z > \pi/2$
- For matched beam, $k_p^2 = k_\beta^2$ half of the beam wave-breaks!
- Stay away from equilibrium! When $k_p^2 >> k_\beta^2$ there is little wavebreaking, and irreversible emittance growth avoided.

Extension to cylindrical symmetry: 1D simulations



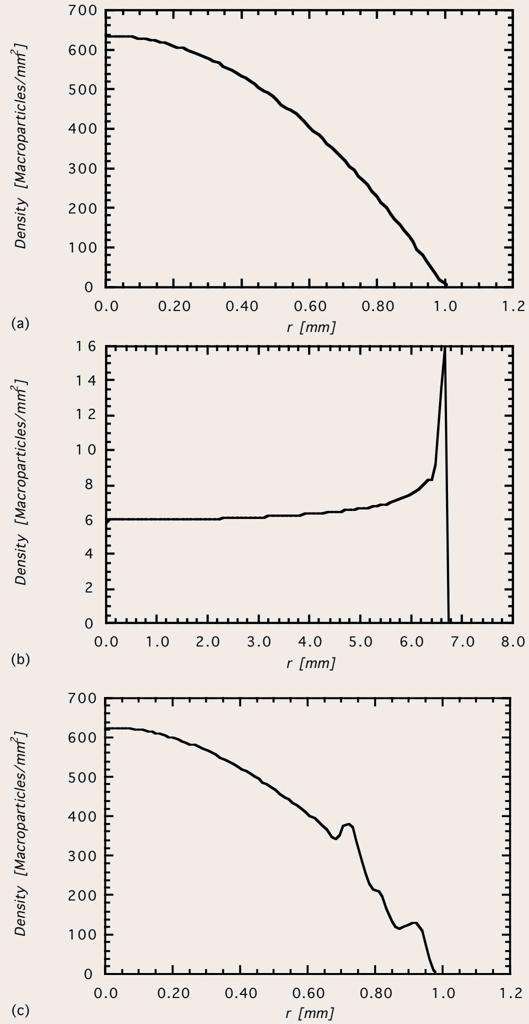


RMS beam size in red, emittance in blue

- Matched parabolic beam shows irreversible emittance growth after single betatron period
- Grossly mismatched single thin lens show excellent *nonlinear* compensation
- Explanation for robustness of first compensation behavior

Emittance growth and entropy

- Irreversible emittance growth is accompanied by entropy increase
- Far-from-equilibrium thin-lens case shows large distortion at beam maximum, near perfect reconstruction of initial profile
- Small wave-breaking region in beam edge

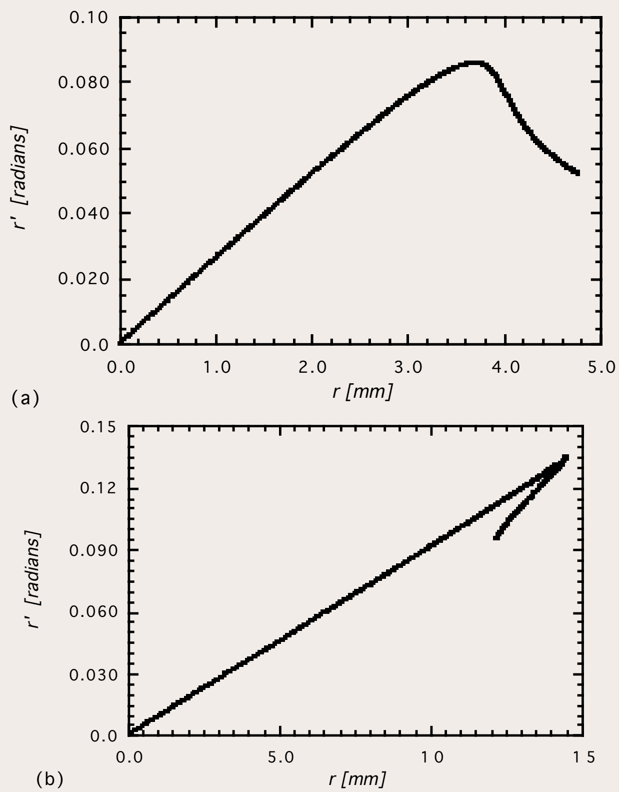


Beam profile at launch

Beam profile at maximum

Beam profile, return to min.

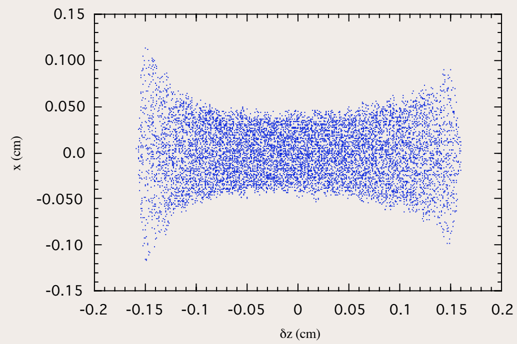
Trace space picture



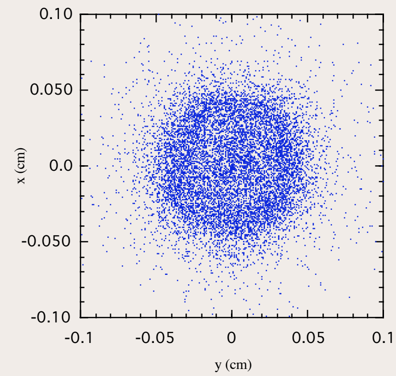
Trace space plots of a freely expanding, initially Gaussian beam at the initial emittance (a) maximum and (b) minimum.

- Wave-breaking occurs near beam edge at emittance maximum
- Fortuitous folding in trace space near "fixed" point minimizes final emittance

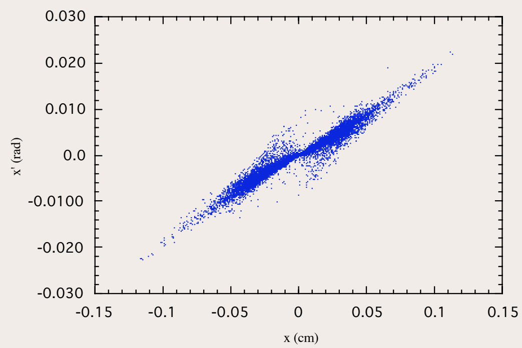
Multiparticle simulation picture: LCLS case (Ferrario scenario)



Spatial (*x-z*) distribution



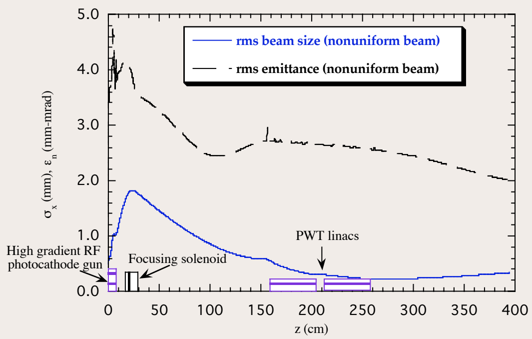
Spatial (*x-y*) distribution



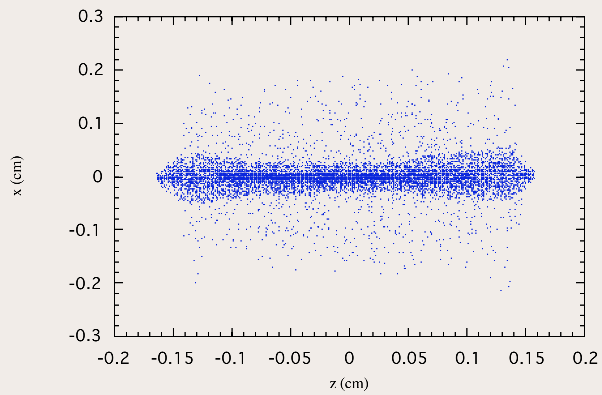
Trace-space distribution

- Case I: initially uniform beam (in *r* and *t*)
- Spatial uniformity reproduced after compensation
- High quality phase space
- Most emittance is in beam longitudinal tails (end effect)

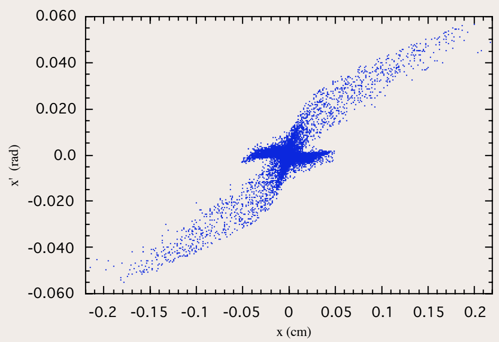
Multiparticle simulation picture: Nonuniform beam



Larger emittance obtained



Spatial (*x*-*z*) distribution



Trace-space distribution

- Case II: Gaussian beam
- Most emittance growth due to nonlinearity
- Large halo formation

The big picture: scaling of design parameters in photoinjectors

- The “beam-plasma” picture based on envelopes gives rise to powerful scaling laws
- RF acceleration also amenable to scaling
- Scale designs with respect to:
  - Charge
  - RF wavelength
- Change from low charge (FEL) to high charge (HEP, wakefield accelerator) design
- Change RF frequency from one laboratory to another (*e.g.* SLAC X-band, TESLA L-band)

Charge scaling

- Keep all accelerator/focusing parameters identical
- To keep plasma the same, the density and aspect ratio of the bunch must be preserved

$$\sigma_i \propto Q^{1/3}$$

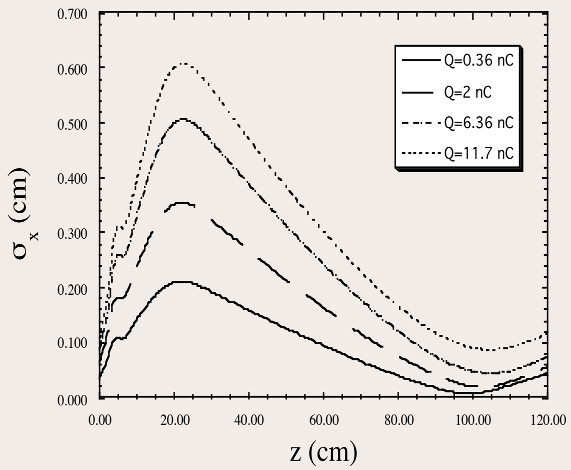
- The contributions to the emittance scale with varying powers of the beam size
- Space-charge emittance $\varepsilon_{x,sc} \propto k_p^2 \sigma_x^2 \propto Q^{2/3}$
- RF/chromatic aberration emittance $\varepsilon_{x,RF} \propto \sigma_z^2 \sigma_x^2 \propto Q^{4/3}$
- Thermal emittance $\varepsilon_{x,th} \propto \sigma_x \propto Q^{1/3}$
- Fortuitously, beam is SC dominated, and these emittancs do not affect the beam envelope evolution; compensation is preserved.

Wavelength scaling

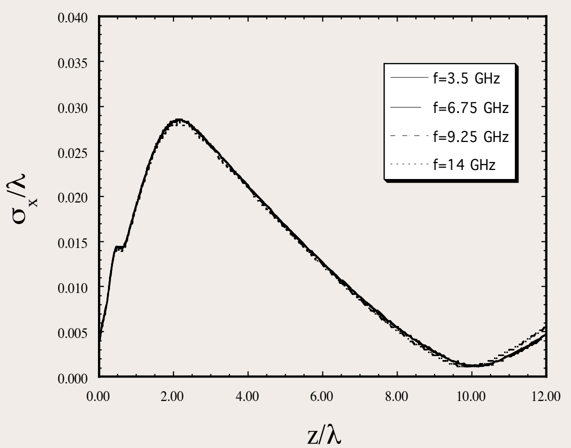
- First, must make acceleration dynamics scale: $\alpha_{RF} \propto E_0\lambda = \text{constant}$ and $E_0 \propto \lambda^{-1}$
- Focusing (betatron) wavenumbers must also scale (RF is naturally scaled, $\lambda_{\beta,RF} \propto E_0$). Solenoid field scales as $B_0 \propto \lambda^{-1}$.
- Correct scaling of beam size, and plasma frequency: $\sigma_i \propto \lambda$ $Q \propto \lambda$
- All emittances scale rigorously as $\varepsilon_n \propto \lambda$

Scaling studies: envelope

- PARMELA simulations used to explore scaling
- Charge scaling (non-optimized case) is only approximate. At large beam charges (beam sizes), beam is not negligibly small compared to RF wavelength.
- Wavelength scaling is exact, as expected.

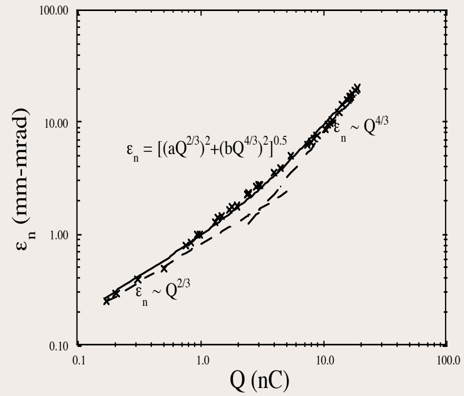


Beam size evolution, different charges

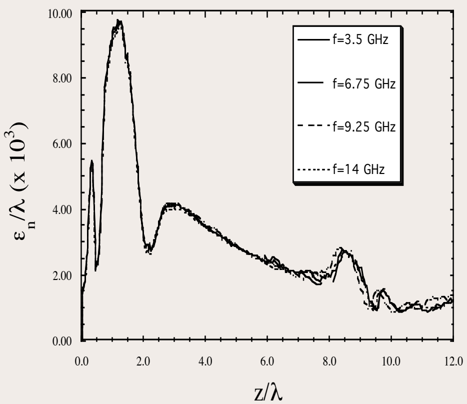


Beam size evolution, different RF λ

Scaling studies: emittance



Scaling of emittance with charge (no thermal emittance), fit assumes addition in squares.



Evolution of emittance, normalized to λ

- Simulation studies verify exact scaling of emittance with λ
- Charge scan of simulations gives information about "family" of designs
- Use to mix scaling laws…

Brightness, choice of charge and wavelength

- Charge and pulse length scale together as λ
- Brightness scales strongly with λ, $B_e = 2I/\varepsilon_n^2 \propto \lambda^{-2}$
- This implies low charge for high brightness
- What if you want to stay at a certain charge (e.g. FEL energy/pulse)
- Mixed scaling:

thermal charge RF/chromatic

$$\varepsilon_n(\text{mm-mrad}) = \lambda_{rf}(\text{m})\sqrt{a_1\left(\frac{Q(\text{nC})}{\lambda_{rf}(\text{m})}\right)^{2/3} + a_2\left(\frac{Q(\text{nC})}{\lambda_{rf}(\text{m})}\right)^{4/3} + a_3\left(\frac{Q(\text{nC})}{\lambda_{rf}(\text{m})}\right)^{8/3}}$$

- For Ferrario scenario, constants from simulation:

$a_1 = 1.5 \qquad a_2 = 0.81 \qquad a_3 = 0.052$

Some practical limits on scaling

- Scaling of beam size
  - laser pulse length and jitter difficult at small λ
  - emittance measurements difficult at small λ
- Scaling of external forces
  - Electric field is "natural" - high-gradient implies short λ because of breakdown limits,.
  - RF limitations may arise in power considerations
  - Focusing solenoid $B \propto \lambda^{-1}$ dimensions scale as λ. Current density scales as $J_{sol} \propto \lambda^{-2}$

Exercises

Problem 5: Assume the LCLS photoinjector has gradient of 20 MV/m, and is run on the invariant envelope with η=1, achieving a normalized emittance of 0.9 mm-mrad at 100 A current. At the final energy of 150 MeV, what is the ratio of the space-charge term to the emittance term in the envelope equation?

Problem 6: (a) For the parameters of the LCLS design family (Ferrario scenario), if one desires to run at 1 nC, what is the optimum RF wavelength to choose to minimize the emittance? (b) If you operate at an RF wavelength of 10.5 cm, what choice of charge maximizes the brightness?